Rossmoyne Senior High School

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

SOLUTIONS

Section One: Calculator-free

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (51 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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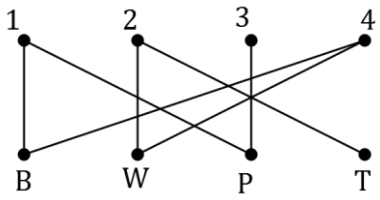
Question 1 **(6 marks)**

(a) A farmer has four fields that are to be used to grow crops of barley, wheat, potatoes and turnips. Field 1 is suitable for barley or potatoes, Field 2 for wheat or turnips, Field 3 for potatoes and Field 4 for barley or wheat.

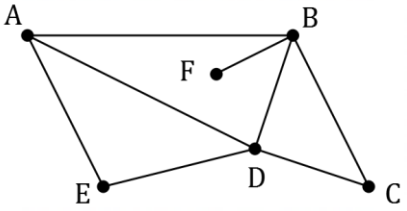
(i) Represent this information using a bipartite graph. (2 marks)

Solution

Specific behaviours
✓ two labelled sets of vertices ✓ correct bipartite graph

(ii) Given that all four crops must be grown at the same time, and only one crop can be grown per field, in which field should the farmer grow the crop of wheat? Justify your answer. (1 mark)

Solution
Field 4. Although fields 2 and 4 are both suitable for wheat, 2 must be used for turnips, leaving 4 for wheat.
Specific behaviours
✓ correct field, with reasonable justification

(b) The graph shown to the right represents unsealed tracks between six barns on the farm.



(i) State the length of the longest cycle in the graph. (1 mark)

Solution
Length is 5.
Specific behaviours
✓ correct length

(ii) Explain why the graph is not Hamiltonian. (1 mark)

Solution
It does not contain a cycle through all six vertices Or goes through all vertices without repeats and finishes at different vertex
Specific behaviours
✓ reasonable explanation

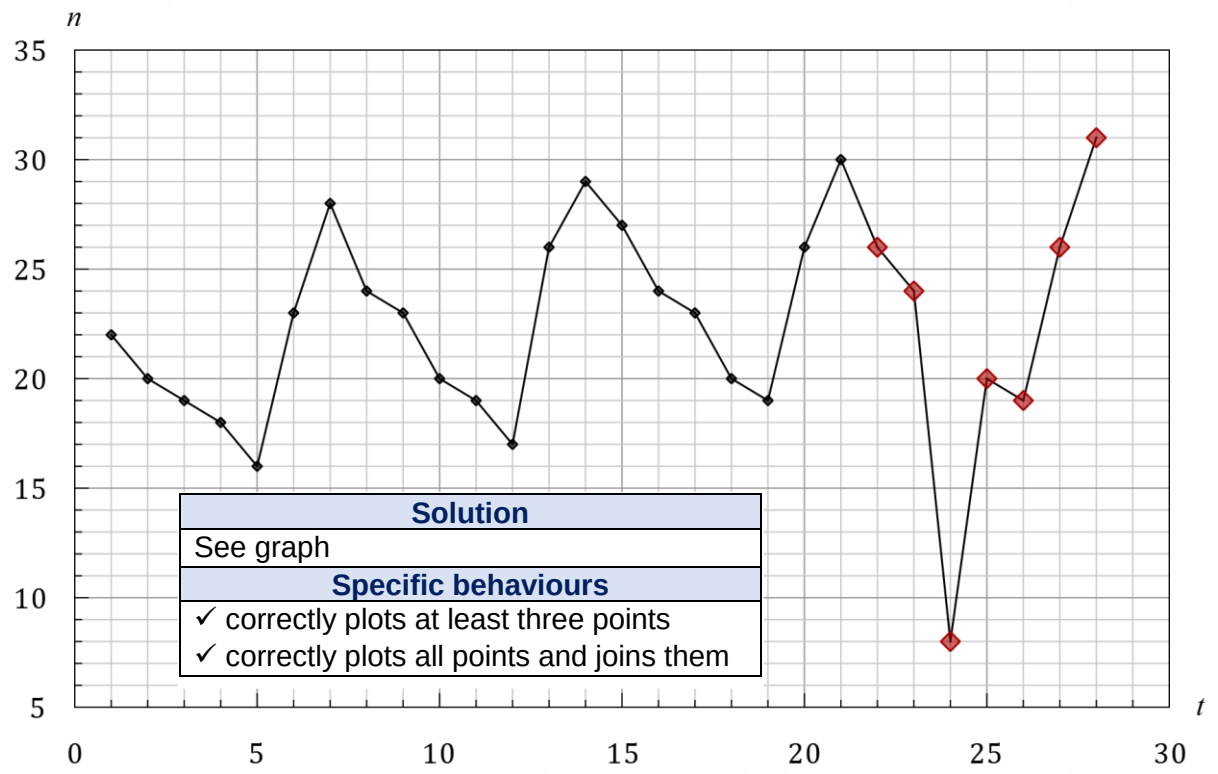
(iii) An edge can be added to the graph so that it becomes Hamiltonian. State a pair of vertices between which such an edge should be added. (1 mark)

Solution
Between F and one of A or C or E.
Specific behaviours
✓ correct pair of vertices

See next page

Question 2 **(5 marks)**

The daily number of gas bottle refills (n) over a period of three weeks at a service station is shown in the plot below. The Monday of Week 1 corresponds to $t = 1$.



Solution
See graph
Specific behaviours
✓ correctly plots at least three points ✓ correctly plots all points and joins them

The number of refills during Week 4 are shown in the table below.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Time, t	22	23	24	25	26	27	28
Refills, n	26	24	8	20	19	26	31

(a) Complete the time series plot by adding the above data for Week 4. (2 marks)

(b) Describe the trend, seasonality and any other features of the time series plot. (3 marks)

Solution
The trend of the plot is increasing. The seasonality has a weekly cycle, as evidenced by the highest number of refills every Sunday and the lowest every Friday, except for a possible outlier on day 24, when refills appear to be unusually low.
Specific behaviours
Any 3 of the below statements, one mark each • states the overall trend as increasing or positive • high and low parts of season by days • refers to likely outlier on day 24 • weekly seasonality or period of 7

See next page

Question 3 **(6 marks)**

A computer is running some image recognition software that is learning a new task.

On the n^{th} run of the software, the time taken to complete the task, T_n seconds, is given by the recurrence relation

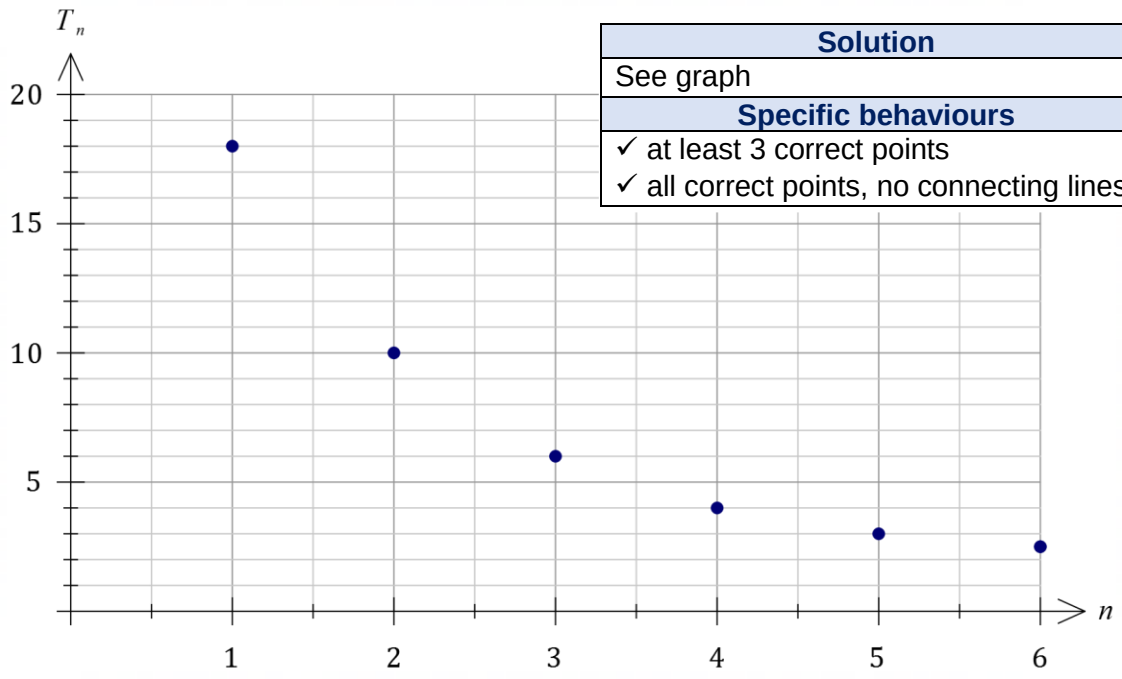
$$T_{n+1} = 0.5T_n + 1, \qquad T_1 = 18.$$

(a) Use the recurrence relation to complete the table below. (2 marks)

n	1	2	3	4	5	6
T_n	**18**	**10**	**6**	**4**	**3**	**2.5**

Solution
See table
Specific behaviours
✓ at least 3 correct entries
✓ all correct entries

(b) Plot the times on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ at least 3 correct points
✓ all correct points, no connecting lines

(c) According to the recurrence relation, the time taken to complete the task will never be quicker than t seconds. Determine, with justification, the value of t. (2 marks)

Solution	
In steady state, $T_{n+1} = T_n = t$. Substituting into recurrence relation: $t = 0.5t + 1$ $0.5t = 1$ $t = 2$ ms	When $T_n = 2$ then $T_{n+1} = 0.5 \times 2 + 1$ $= 2$ Hence steady state when $t = 2$ ms.
Specific behaviours	
✓ any reasonable justification ✓ correct value for t	

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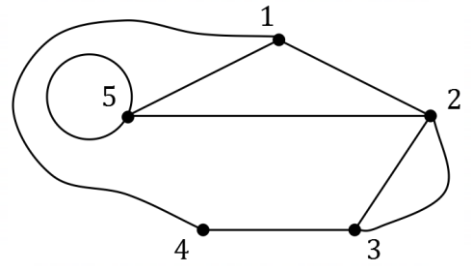
Question 4 **(7 marks)**

An employee at a reticulation company represented three lawn watering systems using graphs G_1, G_2 and G_3.

(a) Graph G_1 has adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$.

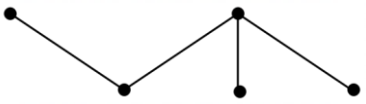
Draw graph G_1 in the plane, labelling the vertices A, B, C, D and E (3 marks)

Solution

Specific behaviours
✓ graph with five vertices, correctly labelled ✓ loop at vertex E ✓ correct graph drawn in the plane (no crossed edges)

(b) Graph G_2 is a connected planar graph with 18 edges and 11 faces.
Determine the number of vertices in graph G_2. (2 marks)

Solution
Using Euler's formula $v + 11 - 18 = 2$. Hence $v = 9$, and so there are 9 vertices in graph G_2.
Specific behaviours
✓ correctly substitutes into Euler's formula ✓ correct number of vertices

(c) Graph G_3 has 5 vertices, is a tree, and the longest trail it contains has length 3.
Draw graph G_3. (2 marks)

Solution

Specific behaviours
✓ tree with 5 vertices ✓ correct graph

See next page

Question 5 **(8 marks)**

Four workers, A, B, C and D, are each to be assigned to one of four tasks, 1, 2, 3 and 4. Each worker must be assigned to one task, and each task must be done by exactly one worker. Worker C cannot be assigned to task 3. The profit, in dollars, that each worker would generate when assigned to each task is shown in the table below.

	1	**2**	**3**	**4**
A	76	51	90	64
B	79	60	79	59
C	71	54	–	62
D	73	56	85	70

(a) Explain two initial modifications that must be made to the table so that the Hungarian algorithm may be applied to the resulting figures in order to determine the maximum total profit that can be generated by the four workers. (2 marks)

Solution
Set the profit for worker C, task 3, to any value less than 54, such as 0. Subtract each entry from 90 (or larger).
Specific behaviours
✓ indicates need for, or states, a suitable dummy value for C3 ✓ indicates need to convert from maximising to minimising problem

(b) In the space provided below, modify the table as required in part (a). (1 mark)

Solution
$\begin{bmatrix} 76 & 51 & 90 & 64 \\ 79 & 60 & 79 & 59 \\ 71 & 54 & 0 & 62 \\ 73 & 56 & 85 & 70 \end{bmatrix} \rightarrow \begin{bmatrix} 14 & 39 & 0 & 26 \\ 11 & 30 & 11 & 31 \\ 19 & 36 & 90 & 28 \\ 17 & 34 & 5 & 20 \end{bmatrix}$
Specific behaviours
✓ suitable value for C3 and subtracts from constant to leave all non-negative values

See next page

(c) It is decided that worker C can be assigned task 3 if required. Using the Hungarian algorithm, determine the maximum total profit and the assignment of workers required to achieve this maximum using the table of the values below. (5 marks)

	1	**2**	**3**	**4**
A	76	51	90	64
B	79	60	79	59
C	71	54	61	62
D	73	56	85	70

Solution

Reduce rows, columns and then apply algorithm to create extra zeros:

$$\begin{bmatrix} 14 & 39 & 0 & 26 \\ 0 & 19 & 0 & 20 \\ 0 & 17 & 10 & 9 \\ 12 & 29 & 0 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 14 & 22 & 0 & 17 \\ 0 & 2 & 0 & 11 \\ 0 & 0 & 10 & 0 \\ 12 & 12 & 0 & \mathbf{6} \end{bmatrix} \rightarrow \begin{bmatrix} 8 & 16 & \mathbf{0} & 11 \\ \mathbf{0} & 2 & 6 & 11 \\ 0 & \mathbf{0} & 16 & 0 \\ 6 & 6 & 0 & \mathbf{0} \end{bmatrix}$$

(NB Zeros in middle table can be covered with 3 lines: rows B and C, and column 3, leaving smallest uncovered number 6).

Required assignment (bolded) is $A \rightarrow 3, B \rightarrow 1, C \rightarrow 2, D \rightarrow 4$.

This assignment generates a maximum profit of $90 + 79 + 54 + 70 = \$293$.

Specific behaviours

✓ correctly reduces rows
✓ correctly reduces columns
✓ correctly creates extra zeros
✓ states assignment
✓ states maximum profit

Alternative

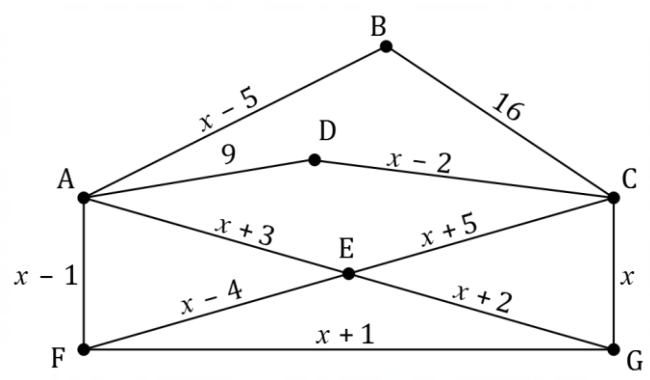
If students cover zeros with 3 lines: row C, and column 1 and 3, leaving smallest uncovered number 2.
Then students will need a 2nd adjustment covering zeros with 3 lines: rows B and C, and column 3, leaving smallest uncovered number 4.

$$\begin{bmatrix} \mathbf{14} & \mathbf{22} & \mathbf{0} & \mathbf{17} \\ \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{11} \\ \mathbf{0} & \mathbf{0} & \mathbf{10} & \mathbf{0} \\ \mathbf{12} & \mathbf{12} & \mathbf{0} & \mathbf{6} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{14} & \mathbf{20} & \mathbf{0} & \mathbf{15} \\ \mathbf{0} & \mathbf{2} & \mathbf{0} & \mathbf{9} \\ \mathbf{0} & \mathbf{0} & \mathbf{12} & \mathbf{0} \\ \mathbf{12} & \mathbf{10} & \mathbf{0} & \mathbf{4} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{10} & \mathbf{16} & \mathbf{0} & \mathbf{11} \\ \mathbf{0} & \mathbf{0} & \mathbf{4} & \mathbf{9} \\ \mathbf{2} & \mathbf{0} & \mathbf{16} & \mathbf{0} \\ \mathbf{8} & \mathbf{6} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

See next page

Question 6 **(8 marks)**

The edges in the graph below represent footpaths between buildings (shown as vertices) on the extensive campus of a research centre. Each edge weight represents the time, in minutes, that a supervisor takes to walk along that footpath to check that it is clean and in good condition.



(a) State, with justification, whether the graph is Eulerian, semi-Eulerian or neither. (2 marks)

Solution
The graph has exactly two odd vertices and so it is semi-Eulerian.
Specific behaviours
✓ states semi-Eulerian, with justification ✓ justification indicating graph has **exactly** two odd vertices (*Do not accept 'has two odd vertices' unless also states that rest are even*)

The supervisor's office is in building G. The least time that the supervisor takes to complete an inspection of all footpaths and return to their office is 115 minutes.

(b) (b) Show algebraically that the value of x, the time to walk along the footpath from C to G is equal to 9. (No marks will be awarded for substituting $x = 9$). (4 marks)

Solution
Semi-Eulerian trail starts at G, ends at F. Then return to G along FG. Sum of edges: $x - 5 + 16 + 9 + x - 2 + x - 1 + x + 3 + x + 5 + x + x - 4 + x + 2 + x + 1 = 9x + 24$ Hence, with repeated edge FG, we get $9x + 24 + x + 1 = 115 \rightarrow 10x = 90 \rightarrow x = 9$.
Specific behaviours
✓ Uses all the edges ✓ adds the edges for an expression ✓ forms an equation ✓ correct value of x

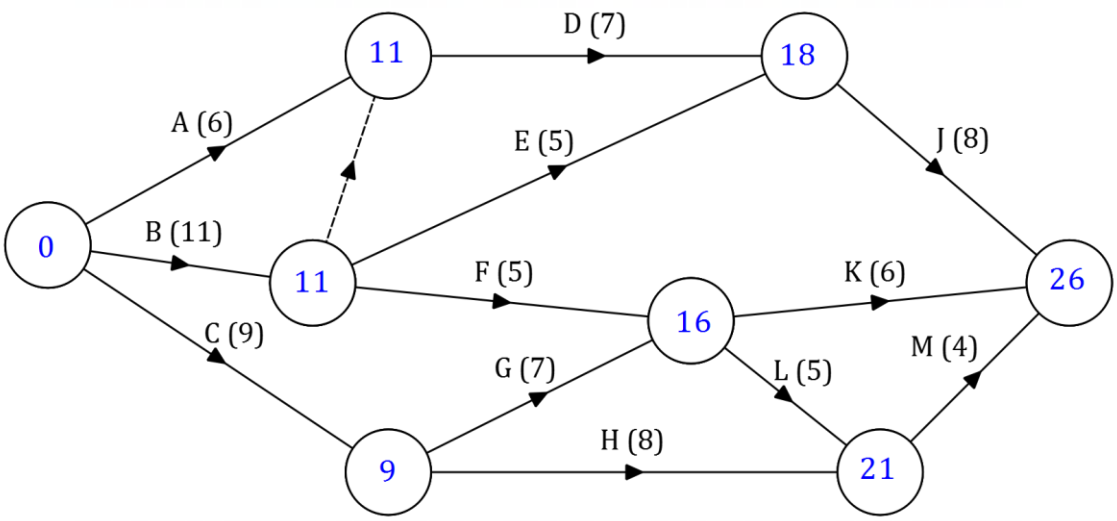
See next page

(c) The supervisor is in their office. At $10{:}45$ am, they are asked to inspect the footpath between buildings A and D. Determine the earliest time that they could finish the inspection of this footpath if they left their office immediately and did not need to return to the office. (2 marks)

Solution
Shortest path to reach one end of edge AD is GCD: $2x - 2 = 16$ minutes. Hence finish inspection $16 + 9 = 25$ minutes after leaving office, at $11{:}10$ am.
Specific behaviours
✓ indicates shortest path to reach one end of edge ✓ correct finish time

Question 7 **(11 marks)**

(a) The network showing the tasks that need to be undertaken to complete a different project is shown below. The duration of each task, in minutes, is shown in brackets.



(i) What does the dotted line on the network indicate? (1 mark)

Solution
Task D cannot start until task B has finished.
Specific behaviours
✓ states correct interpretation

(ii) Determine, in order, the tasks that lie on the critical path and the minimum completion time for the project. (2 marks)

Solution
Tasks on critical path are B, D, J, and the minimum completion time is 26 minutes.
Specific behaviours
✓ correctly lists task on critical path ✓ correctly states the minimum completion time

(iii) Determine the float time for task H. (1 mark)

Solution
$F_H = 26 - 4 - 8 - 9 = 5$ minutes.
Specific behaviours
✓ correct float time

(iv) If task B could be completed in only 8 minutes, how would this affect the critical path and minimum completion time? (2 marks)

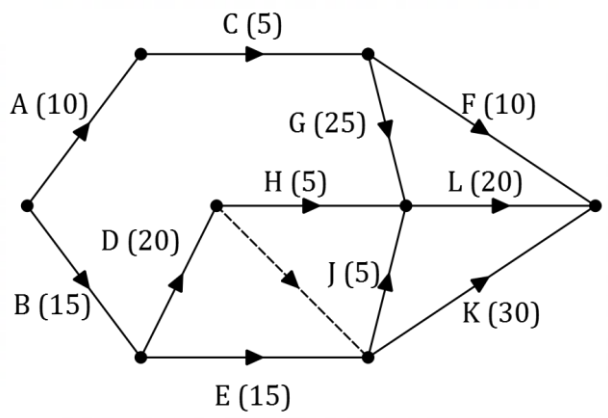
Solution
The tasks on critical path would change to C, G, L, M, and the minimum completion time would be 25 minutes.
Specific behaviours
✓ correctly lists tasks on new critical path ✓ correctly states the new minimum completion time

(b) The tasks required to complete a project are shown in the table below, together with the duration of each task in days, and their immediate predecessor(s).

Task	Duration (days)	Immediate predecessor(s)
A	10	None
B	15	None
C	5	A
D	20	B
E	15	B
F	10	C
G	25	C
H	5	D
J	5	D, E
K	30	D, E
L	20	G, H, J

(i) Construct a project network to represent the information in the table. (4 marks)

Solution



Specific behaviours

✓ correctly add tasks A, B, C, D, E
✓ correctly adds remaining tasks
✓ uses dummy edge to show D must finish before J and K can start
✓ indicates direction and duration of all tasks

(ii) State the earliest time that task G can commence. (1 mark)

Solution

The earliest start time is $10 + 5 = 15$ days.

Specific behaviours

✓ correct time

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

End of questions